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A theoretical analysis of piezoelectric/composite anisotropic laminate with larger-amplitude deflection effect, Part I: Fundamental equations

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Abstract

Considering the effects of both the different material properties of composite layers and the poling directions of piezoelectric layers, we utilized the assumption of the simple-higher-order shear deformation theory to model and analyze the laminated composite plate integrated with the random poled piezoelectric layers. Further, the generalized Hamilton's variation principle for electro-elasticity was employed to deduce the fundamental equations of piezoelectric/composite anisotropic laminate, i.e. the governing equations and boundary conditions. For the special requirement of the larger-amplitude deflection of smart structures, the Von Karman strains were used to account for the geometric nonlinear effect of the practical larger-amplitude deflection on the electro-elastic behavior of smart composite structures. Moreover, the sensor equations were also carried out with considering the large-amplitude deflection effect of smart composite structures.

Keywords: Piezoelectric; Anisotropic; Composite

1. Introduction

In recent decades, the piezoelectric ceramics with a distributed poling direction have been extensively designed as actuators/sensors to apply in the adaptive structures and systems for controlling or

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monitoring the shape deformation of engineering structures due to their excellent piezoelectricity and pyroelectricity after a controllable poling process (Kenji, 1998). However, the early commercial piezoelectric materials were too brittle to evoke their much wider application in some special smart structures require very larger displacement (>1000 μm), such as linear motors, cavity pumps, switches and loud speakers, etc. As the new piezoelectric polymer, including PVDF, PVF_2, etc., with good properties of flexibility, ruggedness, softness and light weight was discovered by Kawai (1969), the disadvantage of the brittle piezoelectric materials in the special requirement smart structures was successfully overcome. Furthermore, the advance of processing technology of thin film and composite in piezoelectric materials also promotes their successful applications in the special requirement structures. Therefore, more and more novel smart structures integrated with the piezoelectric materials have been achieved for the particular demand. For example, Haertling (1994) developed a new-method for producing ultra-high-displacement actuator that can exceed 300% by combining the piezoelectric layer and reduced layer; Lee and Li (1998) established a motor by the new mechanism that was using the extension-twisting coupling of anti-symmetric fiber-reinforced composite laminate induced by the integrated actuator-piezoelectric plate subjected to the external electric field, which bending behavior was further experimentally studied and revealed the large-deflection results by Cheng et al. (2000). Thus, increasing uses of piezoelectric materials in the smart structures and systems call for better understand and accurate analysis of their electrical and mechanical behaviors for the future better design. As is well known, from the viewpoint of piezoelectric work mechanism, it is definitely summarized that all of the piezoelectric actuators and sensors either surface bonded or embedded in the host structures for the adaptive structure system are based on either its extension or shear mechanism. In order to have a good theoretical guide to design the smart structures and systems, many analytical linear models with the classical or first-order shear theory had been developed to describe and predict the electro-mechanical behavior of the different smart structures, involving the one-dimensional, two-dimensional and three-dimensional models. For instance, Crawley and Anderson (1989) and Crawley and de Luis (1987) studied the electroelastical properties of a piezoelectric/elastic laminated beam with its extension mechanism; Lee (1990) incorporated the piezoelectric effect into the classical laminate plate theory to analyze the piezoelectric laminate plate with the extension action of partial electrode covered piezoelectric layer; Zhang and Sun (1996, 1999) utilized the variation principle to derive the governing equations for the adaptive piezoelectric structure using the shear mode of piezoelectric materials and then carried out an approximate solution. Later, other researchers did some similar works on the laminated piezoelectric composite to further explain the linear electro-elastic coupling behavior (Liu et al., 1999; Reddy, 1999; Reddy and Cheng, 2001). However, due to the great difference from the material constants and action between the piezoelectric layers and composite layers in smart structure, it is necessary and important to account for the transverse shear strain of anisotropic piezoelectric/composite laminate. For this case, incorporating the effect of large deflection and in-plane loads caused by piezoelectric layer, Pai et al. (1993) ever only deduced a fully and complicated nonlinear theoretical model for the dynamics and active control of elastic/piezoelectric laminated plate with piezoelectric extension mechanism but not carry out the final numerical results. Thus, establishing a simple 2-D or 3-D theory of piezoelectric/composite plates with piezoelectric extension or shear mechanism to accurately describe the global behavior of smart laminate plate seems to be a compromise between accuracy and ease of analysis. Up to date, few literatures from this view were reported for the smart composite laminated plate with piezoelectric extension or shear mechanism. Therefore, in this paper, we proposed to carry out a set of generalized simple fundamental equations for piezoelectric/composite anisotropic laminated plate with either piezoelectric extension or shear mechanism in the light of Hamilton's variation principle and assumption of Reddy's (1984) simple-high-order theory. This developed analytical model also took the effect of structural larger-deflection and distributed poling direction of the piezoelectric layers into consideration.

2. Theoretical analysis

2.1. Basis assumption and relationship

As shown in Fig. 1, a practical smart structure of piezoelectric/composite laminated plate is generally composed of a number of piezoelectric layers with the spatial poling directions and fiber-reinforced composite laminas with the arbitrary in-plane fiber orientations. In general, the main axial direction of a composite lamina is along the reinforced fiber direction in the x–y plane ($\beta_c = 90°$) with an orientation angle α_c, as shown in Fig. 2(a). Similarly, the main axis (poling direction) of a common piezoelectric layer orients spatially with the distribution angles α_p and β_p from the local coordination system to the global coordination system as shown in Fig. 2(b). Generally, the external electric field is applied to the surface electrode of piezoelectric layer in the z-direction across the thickness. The electro-elastic constitutive relationship of the piezoelectric/composite lamina in the local coordination system can be presented by the stresses σ^l_{ij}, the strains ε^l_{ij}, the electric displacements D^l_i and the electric fields E^l_i in the following formation:

$$\begin{aligned} \sigma^l_{ij} &= C_{ijmn}\varepsilon^l_{mn} - e^{\mathrm{T}}_{kij}E^l_k \\ D^l_i &= e_{imn}\varepsilon^l_{mn} + k_{ij}E^l_j \end{aligned} \tag{1a}$$

where the superscript 'l' denotes those variables in the local coordination system. C_{ij}, e_{ij} and k_{ij} are the stiffness matrix, piezoelectric constant matrix and dielectric constant matrix of the lamina in the local coordination system respectively. It is noted that the piezoelectric constants e_{ij} and dielectric constants k_{ij} are equal to zero inside the composite lamina. After applying the coordination transformed matrices T_c and T_e, as shown in Appendix A, to the above equations for the kth lamina, the lamina constitutive relationship in the global coordination system can be obtained by

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{bmatrix}_k = [T_c]^{-1}_k \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\ C_{12} & C_{22} & C_{23} & 0 & 0 & C_{26} \\ C_{13} & C_{23} & C_{33} & 0 & 0 & C_{36} \\ 0 & 0 & 0 & C_{44} & C_{45} & 0 \\ 0 & 0 & 0 & C_{45} & C_{55} & 0 \\ C_{16} & C_{26} & C_{36} & 0 & 0 & C_{66} \end{bmatrix}_k [T_c]_k \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \varepsilon_{yz} \\ \varepsilon_{zx} \\ \varepsilon_{xy} \end{bmatrix}_k - [T_c]^{-1}_k \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{31} \\ 0 & 0 & e_{33} \\ 0 & e_{15} & 0 \\ e_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_k [T_e]_k \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix}_k \tag{1b}$$

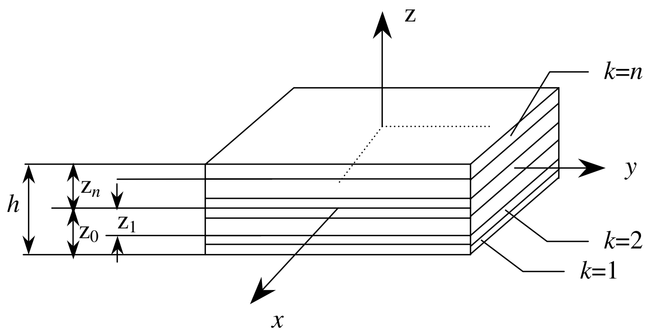

Fig. 1. An illustration diagram for the general piezoelectric/composite laminated plate.

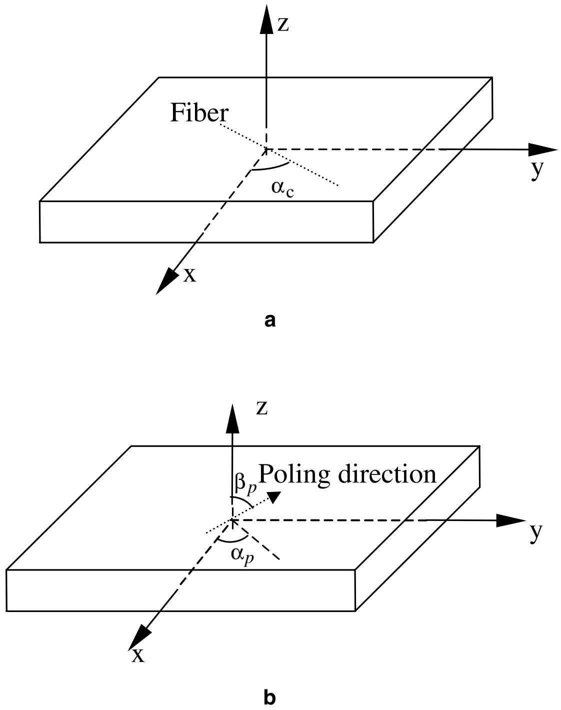

Fig. 2. The schematic show for the main axial distribution of the lamina: (a) composite layer; (b) piezoelectric layer.

$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix}_k = [T_e]_k^{-1} \begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{15} & 0 & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 \end{bmatrix}_k [T_c]_k \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{23} \\ \varepsilon_{31} \\ \varepsilon_{12} \end{Bmatrix}_k + [T_e]_k^{-1} \begin{bmatrix} k_{11} & 0 & 0 \\ 0 & k_{11} & 0 \\ 0 & 0 & k_{33} \end{bmatrix}_k [T_e]_k \begin{Bmatrix} E_1 \\ E_2 \\ E_3 \end{Bmatrix}_k \tag{1c}$$

where the subscript “k” denotes the kth layer and the superscript “-1” means the inverse of the matrix respectively.

In order to consider the effect of larger-amplitude deflection, the Von Karman strains are used to account for the geometric nonlinearities. Then, the relationships between the strains and displacements can be presented by

$$[\varepsilon_x \quad \varepsilon_y \quad \varepsilon_z \quad \varepsilon_{yz} \quad \varepsilon_{zx} \quad \varepsilon_{xy}] = \left[\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2, \frac{\partial u}{\partial y} + \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2, \frac{\partial w_0}{\partial z}, \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial w_0}{\partial y}\frac{\partial w_0}{\partial x}, \frac{\partial w_0}{\partial y} + \frac{\partial v}{\partial z}, \frac{\partial w_0}{\partial x} + \frac{\partial u}{\partial z}\right] \tag{2a}$$

and the electric fields can be induced from the external applied electric potential

$$[E_x, E_y, E_z]^{\mathrm{T}} = -\left[\frac{\partial \varphi}{\partial x}, \frac{\partial \varphi}{\partial y}, \frac{\partial \varphi}{\partial z}\right]^{\mathrm{T}} \tag{2b}$$

In terms of Reddy's simple higher-order theory for the elastic laminated plate, the assumptions of the local elastic displacement field for piezoelectric/composite laminated plate can be also made as

$$\begin{aligned} u &= u_0 + z\left[\phi_x - \frac{4}{3}\left(\frac{z}{h}\right)^2\left(\phi_x + \frac{\partial w_0}{\partial x}\right)\right] \\ v &= v_0 + z\left[\phi_y - \frac{4}{3}\left(\frac{z}{h}\right)^2\left(\phi_y + \frac{\partial w_0}{\partial y}\right)\right] \\ w &= w_0 \end{aligned} \tag{3a}$$

where u_0, v_0 and w_0 denote the displacements of a point (x, y) in the mid-plane. ϕ_x and ϕ_y are the rotations normal to mid-plane about the y and x axes respectively.

In fact, the electric field is always applied along the z-direction across the thickness. Then, due to the existence of piezoelectric layer in smart structure, the electric potential can be taken into consideration and generally assumed as

$$\varphi = \varphi_0 + z\varphi_1 \tag{3b}$$

where φ_0 is the electric potential in mid-plane of the piezoelectric lamina.

Since the piezoelectric lamina can be treated as a thin dielectric plate, it is reasonable to regard φ_0 and φ_1 as constants for the dielectric materials. Then the electric field can be given by

$$[E_x, E_y, E_z]^{\mathrm{T}} = -[0, 0, \varphi_1]^{\mathrm{T}} \tag{3c}$$

The relationship between the applied voltage V_3 and the electric potential φon the electrode is defined as

$$V_3 = \varphi \tag{3d}$$

Substitution of Eq. (3a) into Eq. (2a) can yield the strain–displacement relationship

$$\varepsilon_x = \frac{\partial u_0}{\partial x} + z\left[\phi_{x,x} - \frac{4}{3}\left(\frac{z}{h}\right)^2\left(\phi_{x,x} + \frac{\partial^2 w_0}{\partial x^2}\right)\right] + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2 \tag{4a}$$

$$\varepsilon_y = \frac{\partial v_0}{\partial y} + z\left[\phi_{y,y} - \frac{4}{3}\left(\frac{z}{h}\right)^2\left(\phi_{y,y} + \frac{\partial^2 w_0}{\partial y^2}\right)\right] + \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2 \tag{4b}$$

$$\varepsilon_z = \frac{\partial w_0}{\partial z} \tag{4c}$$

$$\varepsilon_{xy} = \frac{\partial v_0}{\partial x} + \frac{\partial u_0}{\partial y} + \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y} + z\left[\phi_{x,y} - \frac{4}{3}\left(\frac{z}{h}\right)^2\left(\phi_{x,y} + \frac{\partial^2 w_0}{\partial x \partial y}\right)\right] \tag{4d}$$

$$\varepsilon_{yz} = \frac{\partial w_0}{\partial y} + \phi_y - \frac{4z^2}{h^2}\left(\phi_y + \frac{\partial w_0}{\partial y}\right) \tag{4e}$$

$$\varepsilon_{zx} = \frac{\partial w_0}{\partial x} + \phi_x - \left(\frac{2z}{h}\right)^2\left(\phi_x + \frac{\partial w_0}{\partial x}\right) \tag{4f}$$

On the assumption of the strain $\varepsilon_z = 0$ in terms of Reddy's simple-higher-order theory, the strain–displacement relationships, i.e. Eq. (4), can be rewritten by

$$[\,\varepsilon_x \quad \varepsilon_y \quad \varepsilon_z \quad \varepsilon_{xy}\,]^{\mathrm{T}} = \{e1\} + z\{e2\} + z^3\{e4\} \tag{5a}$$

and

$$\left\{\begin{matrix}\varepsilon_{yz} \\ \varepsilon_{zx}\end{matrix}\right\} = \{\tilde{e}1\} + z^2\{\tilde{e}3\} \tag{5b}$$

where the following formulae are used:

$$\{e1\} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial w_0}{\partial x}\right)^2 \\ \dfrac{\partial v_0}{\partial y} + \dfrac{1}{2}\left(\dfrac{\partial w_0}{\partial y}\right)^2 \\ 0 \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} + \dfrac{\partial w_0}{\partial x}\dfrac{\partial w_0}{\partial y} \end{Bmatrix}, \quad \{e2\} = \begin{Bmatrix} \dfrac{\partial \phi_x}{\partial x} \\ \dfrac{\partial \phi_y}{\partial y} \\ 0 \\ \dfrac{\partial \phi_x}{\partial y} + \dfrac{\partial \phi_y}{\partial x} \end{Bmatrix} \tag{6a}$$

$$\{e4\} = -\frac{4}{3h^2}\begin{Bmatrix} \dfrac{\partial \phi_x}{\partial x} \\ \dfrac{\partial \phi_y}{\partial y} \\ 0 \\ \dfrac{\partial \phi_x}{\partial y} + \dfrac{\partial \phi_y}{\partial x} \end{Bmatrix} - \frac{4}{3h^2}\begin{Bmatrix} \dfrac{\partial^2 w_0}{\partial x^2} \\ \dfrac{\partial^2 w_0}{\partial y^2} \\ 0 \\ 2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix} \tag{6b}$$

$$\{\tilde{e}1\} = \begin{Bmatrix} \phi_x + \dfrac{\partial w_0}{\partial x} \\ \phi_y + \dfrac{\partial w_0}{\partial y} \end{Bmatrix}, \quad \{\tilde{e}3\} = -\frac{4}{h^2}\begin{Bmatrix} \phi_x + \dfrac{\partial w_0}{\partial x} \\ \phi_y + \dfrac{\partial w_0}{\partial y} \end{Bmatrix} \tag{6c}$$

In order to derive the equations conveniently, the relative stress resultants can be always defined as follows:

$$\begin{bmatrix} N_x & N_y & N_z & N_{xy} \\ M_x & M_y & M_z & M_{xy} \end{bmatrix} = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \begin{Bmatrix} 1 \\ z \end{Bmatrix} [\,\sigma_x \quad \sigma_y \quad \sigma_z \quad \sigma_{xy}\,]_k \,\mathrm{d}z \tag{7a}$$

$$[\,T_x \quad T_y \quad T_{xy}\,] = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} z^3 [\,\sigma_x \quad \sigma_y \quad \sigma_{xy}\,]_k \,\mathrm{d}z \tag{7b}$$

$$\begin{bmatrix} Q_x & Q_y \\ S_x & S_y \end{bmatrix} = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \begin{Bmatrix} 1 \\ z^2 \end{Bmatrix} [\,\sigma_{xz} \quad \sigma_{yz}\,]_k \,\mathrm{d}z \tag{7c}$$

Substituting Eqs. (1b) and (6) into Eq. (7) can yield

$$\begin{aligned} &[N_x, N_y, 0, N_{xy}]^{\mathrm{T}} = [A_{ij}]e1 + [B_{ij}]e2 + [F_{ij}]e4 - [N_x^p, N_y^p, 0, N_{xy}^p]^{\mathrm{T}} \\ &[M_x, M_y, 0, M_{xy}]^{\mathrm{T}} = [B_{ij}]e1 + [D_{ij}]e2 + [H_{ij}]e4 - [M_x^p, M_y^p, 0, M_{xy}^p]^{\mathrm{T}} \\ &[T_x, T_y, 0, T_{xy}]^{\mathrm{T}} = [F_{ij}]e1 + [H_{ij}]e2 + [J_{ij}]e4 - [T_x^p, T_y^p, 0, T_{xy}^p]^{\mathrm{T}} \\ &[Q_x, Q_y]^{\mathrm{T}} = [A_{ij}]\tilde{e}1 + [D_{ij}]\tilde{e}3 - [Q_x^p, Q_y^p]^{\mathrm{T}} \\ &[S_x, S_y]^{\mathrm{T}} = [D_{ij}]\tilde{e}1 + [H_{ij}]\tilde{e}3 - [S_x^p, S_y^p]^{\mathrm{T}} \end{aligned} \tag{8}$$

where A_{ij}, B_{ij}, etc. are the plate stiffness and defined as follows:

$$[A_{ij}, B_{ij}, D_{ij}, F_{ij}, H_{ij}, J_{ij}] = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} (1, z, z^2, z^3, z^4, z^6) Q_{ij}^k \,\mathrm{d}z \quad (i, j = 1, 2, 6)$$

$$[A_{ij}, D_{ij}, H_{ij}] = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} (1, z^2, z^4) Q_{ij}^k \,\mathrm{d}z \quad (i, j = 4, 5)$$

and N_x^p, N_y^p, etc. denote the stress resultants induced by the coupling electro-elastic behavior of piezoelectric layers due to the external electric fields,

$$[N_x^p, N_y^p, N_{xy}^p] = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} [\bar{e}_{31}^k E_z^k, \bar{e}_{32}^k E_z^k, \bar{e}_{36}^k E_z^k] \mathrm{d}z$$

$$[M_x^p, M_y^p, M_{xy}^p] = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} [\bar{e}_{31}^k E_z^k, \bar{e}_{32}^k E_z^k, \bar{e}_{36}^k E_z^k] z\, \mathrm{d}z$$

$$[T_x^p, T_y^p, T_{xy}^p] = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} [\bar{e}_{31}^k E_z^k, \bar{e}_{32}^k E_z^k, \bar{e}_{36}^k E_z^k] z^3\, \mathrm{d}z$$

$$[Q_x^p, Q_y^p] = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} [\bar{e}_{34}^k E_z^k, \bar{e}_{35}^k E_z^k] \mathrm{d}z$$

$$[S_x^p, S_y^p] = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} [\bar{e}_{34}^k E_z^k, \bar{e}_{35}^k E_z^k] z^2\, \mathrm{d}z$$

with the lamina stiffness and piezoelectric constants matrices $Q_{ij}^k = [T_c]_k^{-1} C_{ij}^k [T_c]_k$ and $\bar{e}_{ij}^k = [T_c]_k^{-1} e_{ij}^k [T_e]_k$. Here, the electric field applied to the piezoelectric layer with partially covered electrode can be described by Heaviside step function as

$$E_z^k = E_0[H(x - x_0) - H(x - x_1)] \times [H(y - y_0) - H(y - y_1)]$$

2.2. The generalized Hamilton variation principle to include electro-elasticity

Under the action of external mechanical and electric field, the simplest form of electric enthalpy which is a compatible with thermodynamics can be given by

$$U = \frac{1}{2} \int (\sigma_{ij}\varepsilon_{ij} - D_i E_i) \mathrm{d}v = \frac{1}{2} \int (C_{ijkl}\varepsilon_{ij}\varepsilon_{kl} - 2e_{ijl}\varepsilon_{jl}E_i - E_j k_{ij} E_i) \mathrm{d}v \tag{9}$$

Integrating with respect of z-direction for Eq. (9) and using Eqs. (7) and (8), the electric enthalpy per unit of piezoelectric/composite laminate can be presented as

$$\begin{aligned} U = \frac{1}{2} \int_{a_1}^{a_2} \int_{b_1}^{b_2} \Big(& [N_x \quad N_y \quad 0 \quad N_{xy}]^{\mathrm{T}} \{e1\} + [M_x \quad M_y \quad 0 \quad M_{xy}]^{\mathrm{T}} \{e2\} + [T_x \quad T_y \quad 0 \quad T_{xy}]^{\mathrm{T}} \{e4\} \\ & + [Q_x \quad Q_y]^{\mathrm{T}} \{\tilde{e}1\} + [S_x \quad S_y]^{\mathrm{T}} \{\tilde{e}3\} - [N_x^p \quad N_y^p \quad 0 \quad N_{xy}^p]^{\mathrm{T}} \{e1\} - [M_x^p \quad M_y^p \quad 0 \quad M_{xy}^p]^{\mathrm{T}} \{e2\} \\ & - [T_x^p \quad T_y^p \quad 0 \quad T_{xy}^p]^{\mathrm{T}} \{e4\} - [Q_x^p \quad Q_y^p] \{\tilde{e}1\} - [S_x^p \quad S_y^p] \{\tilde{e}3\} - \sum_{i=1}^{k} \int_{z_i}^{z_{i+1}} E_z k_{zz} E_z \, \mathrm{d}z \Big) \mathrm{d}x\, \mathrm{d}y \end{aligned} \tag{10}$$

and the kinetic energy is

$$\begin{aligned} T = \frac{1}{2} \int_{a_1}^{a_2} \int_{b_1}^{b_2} \Bigg\{ & I_0 \left[\left(\frac{\partial u_0}{\partial t}\right)^2 + \left(\frac{\partial v_0}{\partial t}\right)^2 + \left(\frac{\partial w_0}{\partial t}\right)^2 \right] + 2I_1 \left(\frac{\partial u_0}{\partial t}\frac{\partial \phi_x}{\partial t} + \frac{\partial v_0}{\partial t}\frac{\partial \phi_y}{\partial t} \right) + I_2 \left[\left(\frac{\partial \phi_x}{\partial t}\right)^2 + \left(\frac{\partial \phi_y}{\partial t}\right)^2 \right] \\ & + 2I_3 \left(\frac{\partial u_0}{\partial t}\frac{\partial \psi_x}{\partial t} + \frac{\partial v_0}{\partial t}\frac{\partial \psi_y}{\partial t} \right) + 2I_4 \left(\frac{\partial \psi_x}{\partial t}\frac{\partial \phi_x}{\partial t} + \frac{\partial \psi_y}{\partial t}\frac{\partial \phi_y}{\partial t} \right) + I_6 \left[\left(\frac{\partial \psi_x}{\partial t}\right)^2 + \left(\frac{\partial \psi_y}{\partial t}\right)^2 \right] \Bigg\} \mathrm{d}x\, \mathrm{d}y \end{aligned} \tag{11}$$

where the rotation inertias are defined by $[I_0, I_1, I_2, I_3, I_4, I_6] = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \rho^{(k)} (1, z, z^2, z^3, z^4, z^6) \mathrm{d}z$ and $\psi_x = -\frac{4}{3h^2}\left(\phi_x + \frac{\partial}{\partial x} w_0\right), \psi_y = -\frac{4}{3h^2}\left(\phi_y + \frac{\partial}{\partial y} w_0\right)$.

Furthermore, the external work done by the applied mechanical load and electrical field is

$$
\begin{aligned}
A=&\int_{a_1}^{a_2}\int_{b_1}^{b_2}\left(\begin{bmatrix} q_x & q_y & q_z\end{bmatrix}\begin{bmatrix} u_0 & v_0 & w_0\end{bmatrix}^{\mathrm{T}}+\begin{bmatrix} m_x & m_y\end{bmatrix}\begin{bmatrix}\phi_x & \phi_y\end{bmatrix}^{\mathrm{T}}+\begin{bmatrix} l_x & l_y\end{bmatrix}\begin{bmatrix}\psi_x & \psi_y\end{bmatrix}^{\mathrm{T}}\right)\mathrm{d}x\,\mathrm{d}y\\
&+\int_{b_1}^{b_2}(N_x u_0+N_{xy}v_0+M_x\phi_x+M_{xy}\phi_y+Q_x w_0+T_x\psi_x+T_{xy}\psi_y)\mathrm{d}y\\
&+\int_{a_1}^{a_2}(N_{xy}u_0+N_y v_0+M_{xy}\phi_x+M_y\phi_y+Q_y w_0+T_{xy}\psi_x+T_y\psi_y)\mathrm{d}x-\int_s\bar{\sigma}\varphi\,\mathrm{d}s
\end{aligned}\tag{12}
$$

where $\bar{\sigma}$ denotes the surface charge of piezoelectric layer. And the new defined stress resultants have the following forms:

$$
\begin{aligned}
&q_x=[\tau_{xz}(h/2)-\tau_{xz}(-h/2)]\\
&q_y=[\tau_{yz}(h/2)-\tau_{yz}(-h/2)]\\
&q=[\sigma_z(h/2)-\sigma_z(-h/2)]\\
&[m_x,l_x]=[\tau_{xz}(h/2)+\tau_{xz}(-h/2)][h/2,h^3/8]\\
&[m_y,l_y]=[\tau_{yz}(h/2)+\tau_{yz}(-h/2)][h/2,h^3/8]
\end{aligned}\tag{13}
$$

In the isothermal process, the generalized Hamilton principle for piezoelectric/composite laminate can be used to derive the dynamics equations at any time interval $[t_1, t_2]$ as

$$
\delta\int_{t_1}^{t_2}(U-A+T)\mathrm{d}t=0 \tag{14}
$$

Finally, substituting Eqs. (10)–(12) into Eq. (14) and applying variational principle to the displacement functions u_0, v_0, w_0, ϕ_x and ϕ_y can yield the governing equations:

$$
\delta u_0:\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}+q_x-I_0\ddot{u}_0-\bar{I}_1\ddot{\phi}_x+\frac{4}{3h^2}I_3\frac{\partial^3 w_0}{\partial x\,\partial t^2}=0 \tag{15a}
$$

$$
\delta v_0:\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}+q_y-I_0\ddot{v}_0-\bar{I}_1\ddot{\phi}_y+\frac{4}{3h^2}I_3\frac{\partial^3 w_0}{\partial y\,\partial t^2}=0 \tag{15b}
$$

$$
\delta\phi_x:\frac{\partial M_x}{\partial x}+\frac{\partial M_{xy}}{\partial y}-Q_x+\frac{4}{h^2}S_x-\frac{4}{3h^2}\left(\frac{\partial T_x}{\partial x}+\frac{\partial T_{xy}}{\partial y}\right)+\frac{2}{3}m_x-I_0\ddot{u}_0-\bar{I}_1\ddot{\phi}_x+\frac{4}{3h^2}\bar{I}_4\frac{\partial^3 w_0}{\partial x\,\partial t^2}=0 \tag{15c}
$$

$$
\delta\phi_y:\frac{\partial M_{xy}}{\partial x}+\frac{\partial M_y}{\partial y}-Q_y+\frac{4}{h^2}S_y-\frac{4}{3h^2}\left(\frac{\partial T_{xy}}{\partial x}+\frac{\partial T_y}{\partial y}\right)+\frac{2}{3}m_y-I_1\ddot{v}_0-\bar{I}_2\ddot{\phi}_y+\frac{4}{3h^2}\bar{I}_4\frac{\partial^3 w_0}{\partial y\,\partial t^2}=0 \tag{15d}
$$

$$
\begin{aligned}
\delta w_0:&\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}+\frac{4}{3h^2}\left(\frac{\partial^2 T_x}{\partial x^2}+\frac{2\partial^2 T_{xy}}{\partial x\,\partial y}+\frac{\partial^2 T_y}{\partial y^2}\right)-\frac{4}{h^2}\left(\frac{\partial S_x}{\partial x}+\frac{\partial S_y}{\partial y}\right)+\frac{\partial}{\partial x}\left(N_x\frac{\partial w_0}{\partial x}+N_{xy}\frac{\partial w_0}{\partial y}\right)\\
&+\frac{\partial}{\partial y}\left(N_{xy}\frac{\partial w_0}{\partial x}+N_y\frac{\partial w_0}{\partial y}\right)+q_z-\frac{1}{3}\left(\frac{\partial m_x}{\partial x}+\frac{\partial m_y}{\partial y}\right)-I_0\ddot{w}_0+I_6\left(\frac{4}{3h^2}\right)^2\frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w_0}{\partial x^2}+\frac{\partial^2 w_0}{\partial y^2}\right)\\
&-\frac{4}{3h^2}I_3\frac{\partial^2}{\partial t^2}\left(\frac{\partial u_0}{\partial x}+\frac{\partial v_0}{\partial y}\right)-\frac{4}{3h^2}\bar{I}_4\frac{\partial^2}{\partial t^2}\left(\frac{\partial\phi_x}{\partial x}+\frac{\partial\phi_y}{\partial y}\right)=0
\end{aligned}\tag{15e}
$$

where the new rotation inertias are given by

$$
\bar{I}_1=I_1-\frac{4}{3h^2}I_3,\quad \bar{I}_4=I_4-\frac{4}{3h^2}I_6,\quad \bar{I}_2=I_2-\frac{8}{3h^2}I_4+\frac{16}{9}I_6,\quad l_x=\frac{h^2}{4}m_x,\quad l_y=\frac{h^2}{4}m_y
$$

and the variation for ψ_x and ψ_y are used during the deducing process as follows:

$$\delta\psi_x = -\frac{4}{3h^2}\left(\delta\phi_x + \frac{\partial}{\partial x}\delta w_0\right), \quad \delta\psi_y = -\frac{4}{3h^2}\left(\delta\phi_y + \frac{\partial}{\partial y}\delta w_0\right)$$

From the variation principle, it is evident that along anyone boundary edge, the variations as following:

$$\begin{aligned} &N_n \text{ or } u_n; \quad N_{ns} \text{ or } u_{ns} \\ &M_n \text{ or } \phi_n; \quad M_{ns} \text{ or } \phi_{ns} \\ &T_n \text{ or } \frac{\partial w_0}{\partial n}; \quad Q_n \text{ or } w_0 \end{aligned} \tag{16a}$$

must satisfy the prescribed boundary condition. Here, the following definitions have been used as

$$\begin{aligned} &u_n = u_0 n_x + v_0 n_y; \quad u_{ns} = -u_0 n_y + v_0 n_x \\ &N_n = N_x n_x^2 + N_y n_y^2 + 2N_{xy} n_x n_y; \quad N_{ns} = (N_y - N_x) n_x n_y + N_{xy}(n_x^2 - n_y^2) \\ &M_n = \widetilde{M}_x n_x^2 + \widetilde{M}_y n_y^2 + 2\widetilde{M}_{xy} n_x n_y; \quad M_{ns} = (\widetilde{M}_y - \widetilde{M}_x) n_x n_y + \widetilde{M}_{xy}(n_x^2 - n_y^2) \\ &Q_n = \widetilde{Q}_x n_x + \widetilde{Q}_y n_y - \frac{4}{3h^2}\frac{\partial T_{ns}}{\partial s} \\ &\widetilde{M}_i = M_i - \frac{4}{3h^2} T_i; \quad \widetilde{Q}_i = Q_i - \frac{4}{3h^2} S_i \\ &\frac{\partial}{\partial n} = n_x \frac{\partial}{\partial x} + n_y \frac{\partial}{\partial s}; \quad \frac{\partial}{\partial s} = n_x \frac{\partial}{\partial y} - n_y \frac{\partial}{\partial x} \end{aligned} \tag{16b}$$

and T_{ns} and T_n are defined in a same manner to N_n and N_{ns} respectively.

Substitution of Eqs. (6) and (8) into Eq. (15) can obtain the governing equations simply presented by the functions of mid-plane displacements and rotations by

$$[a] + [b] + [c] + [d] + [e] + [f] = 0 \tag{17}$$

where the matrices $[a]$, $[b]$, $[c]$, $[d]$, $[e]$ and $[f]$ denote the different parts as following significance respectively.

The term related to the small-deformation is $[a]$,

$$[a] = [L_{ij}]_{5\times5}[u_0, v_0, \phi_x, \phi_y, w_0]^{\mathrm{T}} \tag{18}$$

where $[L_{ij}]$ indicates the linear differential operator matrix and is presented in Appendix B.

The term to the applied mechanical loading is $[b]$,

$$[b] = \left\{ q_x, q_y, \frac{2}{3}m_x, \frac{2}{3}m_y, -q_z + \frac{1}{3}\left(\frac{\partial m_x}{\partial x} + \frac{\partial m_y}{\partial y}\right) \right\}^{\mathrm{T}} \tag{19}$$

The term related to the piezoelectric lamina under the action of electrical field is $[c]$,

$$[c] = -\left\{ \begin{array}{c} \dfrac{\partial N_x^p}{\partial x} + \dfrac{\partial N_{xy}^p}{\partial y} \\ \dfrac{\partial N_{xy}^p}{\partial x} + \dfrac{\partial N_y^p}{\partial y} \\ \dfrac{\partial M_x^p}{\partial x} + \dfrac{\partial M_{xy}^p}{\partial y} - Q_x^p - \dfrac{4}{3h^2} S_x^p - \dfrac{4}{3h^2}\left(\dfrac{\partial T_x^p}{\partial x} + \dfrac{\partial T_{xy}^p}{\partial y}\right) \\ \dfrac{\partial M_{xy}^p}{\partial x} + \dfrac{\partial M_y^p}{\partial y} - Q_y^p - \dfrac{4}{3h^2} S_y^p - \dfrac{4}{3h^2}\left(\dfrac{\partial T_{xy}^p}{\partial x} + \dfrac{\partial T_y^p}{\partial y}\right) \\ \dfrac{\partial Q_x^p}{\partial x} + \dfrac{\partial Q_y^p}{\partial y} - \dfrac{4}{3h^2}\left(\dfrac{\partial^2 T_x^p}{\partial x^2} + \dfrac{\partial^2 T_{xy}^p}{\partial x \partial y} + \dfrac{\partial^2 T_y^p}{\partial y^2}\right) - \dfrac{4}{h^2}\left(\dfrac{\partial S_x^p}{\partial x} + \dfrac{\partial S_{xy}^p}{\partial y}\right) \end{array} \right\} \tag{20}$$

The term to the finite deformation is $[d]$,

$$[d] = \begin{Bmatrix} \dfrac{\partial w_0}{\partial x} L_{1,1} w_0 + \dfrac{\partial w_0}{\partial y} L_{1,2} w_0 \\ \dfrac{\partial w_0}{\partial x} L_{1,2} w_0 + \dfrac{\partial w_0}{\partial y} L_{2,2} w_0 \\ \dfrac{\partial w_0}{\partial x} L_{1,3} w_0 + \dfrac{\partial w_0}{\partial y} L_{2,3} w_0 \\ \dfrac{\partial w_0}{\partial x} L_{1,4} w_0 + \dfrac{\partial w_0}{\partial y} L_{2,4} w_0 \\ \dfrac{\partial w_0}{\partial x} L_{1,5} w_0 + \dfrac{\partial w_0}{\partial y} L_{2,5} w_0 \end{Bmatrix} \tag{21}$$

The term to the in-plane mechanics in z-direction is $[e]$,

$$[e] = -\left\{0, 0, 0, 0, \frac{\partial}{\partial x}\left(N_x \frac{\partial w_0}{\partial x} + N_{xy} \frac{\partial w_0}{\partial y}\right) + \frac{\partial}{\partial y}\left(N_{xy} \frac{\partial w_0}{\partial x} + N_y \frac{\partial w_0}{\partial y}\right)\right\}^{\mathrm{T}} \tag{22}$$

Noted here, when N_x, N_y and N_{xy} are constants, i.e. $N_x = N_x^0,\ N_y = N_y^0$ and $N_{xy} = N_{xy}^0$, the fifth element of the matrix $[e]$ can be re-written as $N_x^0 \frac{\partial^2 w_0}{\partial x^2} + 2N_{xy}^0 \frac{\partial^2 w_0}{\partial x \partial y} + N_y^0 \frac{\partial^2 w_0}{\partial y^2}$. Otherwise, the relative stiffness and displacement must be taken place of N_x, N_y and N_{xy} in terms of the constitutive relationship, i.e. Eqs. (6) and (8).

The motion term is $[f]$,

$$[f] = -\begin{Bmatrix} I_0 \ddot{u}_0 + \bar{I}_1 \ddot{\phi}_x - \dfrac{4}{3} I_3 \dfrac{\partial^3 w_0}{\partial x \partial t^2} \\ I_0 \ddot{v}_0 + \bar{I}_1 \ddot{\phi}_y - \dfrac{4}{3} I_3 \dfrac{\partial^3 w_0}{\partial y \partial t^2} \\ \bar{I}_1 \ddot{u}_0 + \bar{I}_2 \ddot{\phi}_x - \dfrac{4}{3} \bar{I}_4 \dfrac{\partial^3 w_0}{\partial y \partial t^2} \\ \bar{I}_1 \ddot{v}_0 + \bar{I}_2 \ddot{\phi}_y - \dfrac{4}{3} \bar{I}_4 \dfrac{\partial^3 w_0}{\partial y \partial t^2} \\ \left[-I_0 \ddot{w}_0 + \left(\dfrac{4}{3h^2}\right)^2 I_6 \frac{\partial^2}{\partial t^2}\left(\frac{\partial^2 w_0}{\partial x^2} + \frac{\partial^2 w_0}{\partial y^2}\right) \right. \\ \left. - \dfrac{4}{3h^2} I_3 \dfrac{\partial^3}{\partial t^2}\left(\dfrac{\partial u_0}{\partial t} + \dfrac{\partial v_0}{\partial t}\right) - \dfrac{4}{3h^2} \bar{I}_4 \dfrac{\partial^2}{\partial t^2}\left(\dfrac{\partial \phi_x}{\partial x} + \dfrac{\partial \phi_y}{\partial y}\right) \right] \end{Bmatrix} \tag{23}$$

From the fundamental equation (17), it is easy to obtain the simple expression for the piezoelectric/composite laminated plate in either static or dynamic case to analyze their static or dynamic characteristics.

2.3. Analysis of the actuating and sensing function for piezoelectric lamina

While the piezoelectric layer is utilized as an actuator in smart structures and systems, it will generate distributed stress resultants treated as the thermal stress resultants in this paper under the application of electric field. In order to study the interaction between the applied electric field and piezoelectric laminated composite plate with distributed covered electrode in piezoelectric layers, the generalized function i.e. Heaviside step function $H(x)$ and Dirac delta function $\delta(x)$ should be used to describe the distributed covered electrode of piezoelectric layer. Thus, for conveniently studying the interaction, the applied electric field can be rewritten by

$$E_3 = -\varphi(t)F(x,y) \tag{24}$$

where $\varphi(t)$ represents the time function of the applied electric field. $F(x, y)$ denotes the effective electrode using the Heaviside function as introduced by Lee (1990)

$$F(x,y) = [H(x-x_1) - H(x-x_2)] \times [H(y-y_1) - H(y-y_2)] \tag{25}$$

Further, the differential of the function $F(x, y)$ can be defined by

$$\frac{\partial F(x,y)}{\partial x} = [\delta(x-x_1) - \delta(x-x_2)] \times [H(y-y_1) - H(y-y_2)] \tag{26a}$$

$$\frac{\partial F(x,y)}{\partial y} = [H(x-x_1) - H(x-x_2)] \times [\delta(y-y_1) - \delta(y-y_2)] \tag{26b}$$

$$\frac{\partial^2 F(x,y)}{\partial x^2} = [\delta'(x-x_1) - \delta'(x-x_2)] \times [H(y-y_1) - H(y-y_2)] \tag{26c}$$

$$\frac{\partial^2 F(x,y)}{\partial y^2} = [H(x-x_1) - H(x-x_2)] \times [\delta'(y-y_1) - \delta'(y-y_2)] \tag{26d}$$

Now, the effect of a piezoelectric layer as an actuator in the smart structures and systems, i.e. $[c]$, can be capable of Eq. (20).

On the contrast, the piezoelectric layer can be also as a sensor to sense and monitor the shape deformation of the smart structures and systems under the action mechanical loading through the closed-circuit output charge signal. Based on the Gauss's law, the charge $q(t)$ and electric current $i(t)$ of the piezoelectric lamina can be simulated by

$$q(t) = \int_S D_3 \,\mathrm{d}s \tag{27a}$$

$$i(t) = \frac{\mathrm{d}p(t)}{\mathrm{d}t} \tag{27b}$$

where S denotes the area of the covered electrode.

Substituting Eqs. (1c), (5) and (6) into Eq. (27a) can yield

$$\begin{aligned}
q_k(t) &= \frac{1}{2}\left[\int\int_{s(z=z_k)} (\bar{e}^k_{31}\varepsilon_x + \bar{e}^k_{32}\varepsilon_y + \bar{e}^k_{34}\varepsilon_{xy} + \bar{e}^k_{35}\varepsilon_{zy} + \bar{e}^k_{36}\varepsilon_{zx})\mathrm{d}x\,\mathrm{d}y\right.\\
&\quad \left. + \int\int_{s(z=z_{k-1})} (\bar{e}^k_{31}\varepsilon_x + \bar{e}^k_{32}\varepsilon_y + \bar{e}^k_{34}\varepsilon_{xy} + \bar{e}^k_{35}\varepsilon_{zy} + \bar{e}^k_{36}\varepsilon_{zx})\mathrm{d}x\,\mathrm{d}y\right]\\
&= \int\int_S \left\{\bar{e}^k_{31}\left[\frac{\partial u_0}{\partial x} + \frac{1}{2}\left(\frac{\partial w_0}{\partial x}\right)^2\right] + \bar{e}^k_{32}\left[\frac{\partial v_0}{\partial y} + \frac{1}{2}\left(\frac{\partial w_0}{\partial y}\right)^2\right] + \bar{e}^k_{34}\left[\frac{\partial v_0}{\partial x} + \frac{\partial u_0}{\partial y} + \frac{\partial w_0}{\partial x}\frac{\partial w_0}{\partial y}\right]\right.\\
&\quad + \bar{e}^k_{35}\left(\phi_x + \frac{\partial w_0}{\partial x}\right) + \bar{e}^k_{36}\left(\phi_y + \frac{\partial w_0}{\partial y}\right) + \frac{z_k + z_{k-1}}{2}\left[\bar{e}^k_{31}\frac{\partial \phi_x}{\partial x} + \bar{e}^k_{32}\frac{\partial \phi_y}{\partial y} + \bar{e}^k_{34}\left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x}\right)\right]\\
&\quad - \frac{2(z_k^2 + z_{k-1}^2)}{h^2}\left[\bar{e}^k_{35}\left(\phi_x + \frac{\partial w_0}{\partial x}\right) + \bar{e}^k_{36}\left(\phi_y + \frac{\partial w_0}{\partial y}\right)\right]\\
&\quad \left. - \frac{2(z_k^3 + z_{k-1}^3)}{3h^2}\left[\bar{e}^k_{31}\left(\frac{\partial \phi_x}{\partial x} + \frac{\partial^2 w_0}{\partial x^2}\right) + \bar{e}^k_{32}\left(\frac{\partial \phi_y}{\partial y} + \frac{\partial^2 w_0}{\partial y^2}\right) + \bar{e}^k_{34}\left(\frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} + 2\frac{\partial^2 w_0}{\partial x\,\partial y}\right)\right]\right\}\mathrm{d}x\,\mathrm{d}y
\end{aligned} \tag{28}$$

So, the closed circuit sensor equation can be also rewritten as

$$
\begin{aligned}
i_k(t) = \int\int_S \Bigg\{ & \bar{e}_{31}^k \left[\frac{\partial^2 u_0}{\partial x\,\partial t} + \frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial x\,\partial t}\right] + \bar{e}_{32}^k \left[\frac{\partial^2 v_0}{\partial y\,\partial t} + \frac{\partial w_0}{\partial y}\frac{\partial^2 w_0}{\partial y\,\partial t}\right] \\
& + \bar{e}_{34}^k \left[\frac{\partial^2 v_0}{\partial x\,\partial t} + \frac{\partial^2 u_0}{\partial y\,\partial t} + \frac{\partial^2 w_0}{\partial x\,\partial t}\frac{\partial w_0}{\partial y} + \frac{\partial w_0}{\partial x}\frac{\partial^2 w_0}{\partial y\,\partial t}\right] + \bar{e}_{35}^k \left(\frac{\partial \phi_x}{\partial t} + \frac{\partial^2 w_0}{\partial x\,\partial t}\right) + \bar{e}_{36}^k \left(\frac{\partial \phi_y}{\partial t} + \frac{\partial^2 w_0}{\partial y\,\partial t}\right) \\
& + \frac{z_k + z_{k-1}}{2}\left[\bar{e}_{31}^k \frac{\partial^2 \phi_x}{\partial x\,\partial t} + \bar{e}_{32}^k \frac{\partial^2 \phi_y}{\partial y\,\partial t} + \bar{e}_{34}^k \left(\frac{\partial^2 \phi_x}{\partial y\,\partial t} + \frac{\partial^2 \phi_y}{\partial x\,\partial t}\right)\right] - \frac{2(z_k^2 + z_{k-1}^2)}{h^2}\left[\bar{e}_{35}^k \left(\frac{\partial \phi_x}{\partial t} + \frac{\partial^2 w_0}{\partial x\,\partial t}\right)\right. \\
& \left. + \bar{e}_{36}^k \left(\frac{\partial \phi_y}{\partial t} + \frac{\partial^2 w_0}{\partial y\,\partial t}\right)\right] - \frac{2(z_k^3 + z_{k-1}^3)}{3h^2}\left[\bar{e}_{31}^k \left(\frac{\partial^2 \phi_x}{\partial x\,\partial t} + \frac{\partial^3 w_0}{\partial x^2\,\partial t}\right) + \bar{e}_{32}^k \left(\frac{\partial^2 \phi_y}{\partial y\,\partial t} + \frac{\partial^3 w_0}{\partial y^2\,\partial t}\right)\right. \\
& \left. + \bar{e}_{34}^k \left(\frac{\partial^2 \phi_x}{\partial y\,\partial t} + \frac{\partial^2 \phi_y}{\partial x\,\partial t} + 2\frac{\partial^3 w_0}{\partial x\,\partial y\,\partial t}\right)\right]\Bigg\} \mathrm{d}x\,\mathrm{d}y
\end{aligned}
\tag{29}
$$

Up to now, the complete theoretical analysis for piezoelectric/composite anisotropic laminate has been established. In Part II (Cheng et al., 2005), we will apply the above theoretical analysis model to study some detailed examples and further carry out the numerical results by a new developed numerical method.

3. Conclusion

Based on Hamilton's variation principle for electro-elasticity, the generalized governing equations and relative boundary conditions for the anisotropic piezoelectric/composite laminate plate were carried out on the assumption of Reddy's simple high-order theory. Here, the effects of the poling direction of piezoelectric layers and large amplitude deflection were taken into consideration. Furthermore, considering the action of piezoelectric layer as a sensor, the sensing equation was also deduced in this paper with the effect of large-amplitude deflection.

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Appendix A

As shown in Fig. 2(b), the principal axe x_3 of a crystal with an eigenfield may be not parallel to the fixed coordinate system. Then, we introduce a local coordinate system (x_1^L, x_2^L, x_3^L) to describe the crystal distribution. If it is assumed that the x_3^L lies in (x_1, x_2) plane, the transformation matrix T from the fixed coordinate system to the local one has the following form as

$$
T_{ij} = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha\cos\beta & \cos\alpha\cos\beta & \sin\beta \\ \sin\alpha\sin\beta & -\sin\beta\cos\alpha & \cos\beta \end{bmatrix}
$$

Then the local variables can be presented by the global variables as

$$\varepsilon_{ij}^{l} = T_{im}T_{jn}\varepsilon_{mn}$$

$$E_i^l = T_{in}E_n$$

Further, we can obtain the coordination transfer matrices T_c and T_e as follows:

$$[T_c] = \begin{bmatrix} n^2 & m^2 & 0 & 0 & 0 & 2mn \\ m^2p^2 & n^2p^2 & q^2 & 2npq & -2mpq & -2nmp^2 \\ m^2q^2 & q^2n^2 & p^2 & -2npq & 2mpq & -2mnq^2 \\ pqm^2 & -pqn^2 & qp & np^2 - nq^2 & mq^2 - mp^2 & 2nmpq \\ nmq & -nmq & 0 & mp & np & qm^2 - qn^2 \\ -nmp & nmp & 0 & mq & nq & pn^2 - pm^2 \end{bmatrix}$$

and

$$[T_e] = \begin{bmatrix} n & m & 0 \\ -mp & np & q \\ mq & -nq & p \end{bmatrix}$$

where $n = \cos\alpha, m = \sin\alpha, p = \cos\beta, q = \sin\beta$.

Appendix B

The components of the matrix $[L_{ij}]$ are shown as follows:

$$L_{1,1} = A_{11}\frac{\partial^2}{\partial x^2} + 2A_{16}\frac{\partial^2}{\partial x\partial y} + A_{66}\frac{\partial^2}{\partial y^2}$$

$$L_{1,2} = A_{16}\frac{\partial^2}{\partial x^2} + (A_{12} + A_{66})\frac{\partial^2}{\partial x\partial y} + A_{26}\frac{\partial^2}{\partial y^2}$$

$$L_{1,3} = \left(B_{11} - \frac{4}{3h^2}F_{11}\right)\frac{\partial^2}{\partial x^2} + 2\left(B_{16} - \frac{4}{3h^2}F_{16}\right)\frac{\partial^2}{\partial x\partial y} + \left(B_{66} - \frac{4}{3h^2}F_{66}\right)\frac{\partial^2}{\partial y^2}$$

$$L_{1,4} = \left(B_{16} - \frac{4}{3h^2}F_{16}\right)\frac{\partial^2}{\partial x^2} + \left(B_{12} + B_{66} - \frac{4}{3h^2}F_{12} - \frac{4}{3h^2}F_{66}\right)\frac{\partial^2}{\partial x\partial y} + \left(B_{26} - \frac{4}{3h^2}F_{26}\right)\frac{\partial^2}{\partial y^2}$$

$$L_{1,5} = -\frac{4}{3h^2}\left[F_{11}\frac{\partial^3}{\partial x^3} + 3F_{16}\frac{\partial^3}{\partial y\partial x^2} + (F_{12} + 2F_{66})\frac{\partial^3}{\partial x\partial y^2} + F_{26}\frac{\partial^3}{\partial y^3}\right]$$

$$L_{2,2} = A_{66}\frac{\partial^2}{\partial x^2} + 2A_{26}\frac{\partial^2}{\partial x\partial y} + A_{22}\frac{\partial^2}{\partial y^2}$$

$$L_{2,3} = L_{1,4}$$

$$L_{2,4} = \left(B_{66} - \frac{4}{3h^2}F_{66}\right)\frac{\partial^2}{\partial x^2} + 2\left(B_{26} - \frac{4}{3h^2}F_{26}\right)\frac{\partial^2}{\partial x\partial y} + \left(B_{22} - \frac{4}{3h^2}F_{22}\right)\frac{\partial^2}{\partial y^2}$$

$$L_{2,5} = -\frac{4}{3h^2}\left[F_{16}\frac{\partial^3}{\partial x^3} + (F_{12} + 2F_{66})\frac{\partial^3}{\partial y\partial x^2} + 3F_{26}\frac{\partial^3}{\partial x\partial y^2} + F_{22}\frac{\partial^3}{\partial y^3}\right]$$

$$L_{3,3} = \left(D_{11} - \frac{8}{3h^2}H_{11} + \frac{16}{9h^4}F_{11}\right)\frac{\partial^2}{\partial x^2} + 2\left(D_{16} - \frac{8}{3h^2}H_{16} + \frac{16}{9h^4}J_{16}\right)\frac{\partial^2}{\partial x\partial y}$$
$$+ \left(D_{66} - \frac{8}{3h^2}H_{66} + \frac{16}{9h^4}J_{66}\right)\frac{\partial^2}{\partial y^2} - \left(A_{55} - \frac{8}{h^2}D_{55} + \frac{16}{h^4}H_{55}\right)$$

$$L_{3,4} = \left(D_{16} - \frac{8}{3h^2}H_{16} + \frac{16}{9h^4}T_{16}\right)\frac{\partial^2}{\partial x^2} + \left[D_{12} + D_{66} - \frac{8}{3h^2}(H_{12} + H_{66}) + \frac{16}{9h^4}(J_{12} + J_{66})\right]\frac{\partial^2}{\partial x\,\partial y}$$
$$+ \left(D_{26} - \frac{8}{3h^2}H_{26} + \frac{16}{9h^4}J_{26}\right)\frac{\partial^2}{\partial y^2} - \left(A_{45} - \frac{8}{h^2}D_{45} + \frac{16}{h^4}H_{45}\right)$$

$$L_{3,5} = -\left(A_{55} - \frac{8}{h^2}D_{55} + \frac{16}{h^4}H_{55}\right)\frac{\partial}{\partial x} - \left(A_{45} - \frac{8}{h^2}D_{45} + \frac{16}{h^4}H_{45}\right)\frac{\partial}{\partial y}$$
$$- \frac{4}{3h^2}\left\{\left(H_{11} - \frac{4}{3h^2}J_{11}\right)\frac{\partial^3}{\partial x^3} + 3\left(H_{16} - \frac{4}{3h^2}J_{16}\right)\frac{\partial^3}{\partial x^2\partial y}\right.$$
$$\left. + \left[H_{12} + 2H_{66} - \frac{4}{3h^2}(J_{12} + 2J_{66})\right]\frac{\partial^3}{\partial x\,\partial y^2} + \left(H_{26} - \frac{4}{3h^2}J_{26}\right)\frac{\partial^3}{\partial y^3}\right\}$$

$$L_{4,4} = \left(D_{66} - \frac{8}{3h^2}H_{66} + \frac{16}{9h^4}J_{66}\right)\frac{\partial^2}{\partial x^2} + 2\left(D_{26} - \frac{8}{3h^2}H_{26} + \frac{16}{9h^4}J_{26}\right)\frac{\partial^2}{\partial x\,\partial y}$$
$$+ \left(D_{22} - \frac{8}{3h^2}H_{22} + \frac{16}{9h^4}J_{22}\right)\frac{\partial^2}{\partial y^2} - \left(A_{44} - \frac{8}{h^2}D_{44} + \frac{16}{h^4}H_{44}\right)$$

$$L_{4,5} = -\left(A_{45} - \frac{8}{h^2}D_{45} + \frac{16}{h^4}H_{45}\right)\frac{\partial}{\partial x} - \left(A_{44} - \frac{8}{h^2}D_{44} + \frac{16}{h^4}H_{44}\right)\frac{\partial}{\partial y}$$
$$- \frac{4}{3h^2}\left\{\left(H_{16} - \frac{4}{3h^2}J_{16}\right)\frac{\partial^3}{\partial x^3} + \left[H_{12} + 2H_{66} - \frac{4}{3h^2}(J_{12} + 2J_{66})\right]\frac{\partial^3}{\partial x^2\partial y}\right.$$
$$\left. + 3\left(H_{26} - \frac{4}{3h^2}J_{26}\right)\frac{\partial^3}{\partial x\,\partial y^2} + \left(H_{22} - \frac{4}{3h^2}J_{22}\right)\frac{\partial^3}{\partial y^3}\right\}$$

$$L_{5,5} = -\left(A_{55} - \frac{8}{h^2}D_{55} + \frac{16}{h^4}H_{55}\right)\frac{\partial^2}{\partial x^2} - 2\left(A_{45} - \frac{8}{h^2}D_{45} + \frac{16}{h^4}H_{45}\right)\frac{\partial^2}{\partial y\,\partial x} - \left(A_{44} - \frac{8}{h^2}D_{44} + \frac{16}{h^4}H_{44}\right)\frac{\partial^2}{\partial y^2}$$
$$+ \frac{16}{9h^4}\left[T_{11}\frac{\partial^4}{\partial x^4} + 4T_{16}\frac{\partial^4}{\partial x^3\partial y} + 2(T_{12} + 2T_{66})\frac{\partial^4}{\partial x^2\partial y^2} + 4T_{26}\frac{\partial^4}{\partial x\,\partial y^3} + T_{22}\frac{\partial^4}{\partial y^4}\right]$$

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